Simulations of the inspiral and merger of neutron star binaries

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ScicomP 15 & SP-XXL May 18-22, 2009, Barcelona
Abstract: Binary neutron stars are among the most important sources of gravitational waves and they are also thought to be at the origin of the most catastrophic astrophysical phenomena, namely short gamma-ray bursts. Exploiting our recent breakthroughs in the description of this process, we will use MareNostrum to perform a series of simulations in full general relativistic hydrodynamics of unequal-mass neutron stars binaries during the last stages of their inspiral, merger and over to formation of a black hole surrounded by a hot, high-density torus. We will concentrate on the impact that different initial masses, mass ratios and separations have on the gravitational waves emitted and on the properties of the torus around the rapidly rotating black hole. All the simulations will make use of the codes Whisky/Cactus/Carpet developed at the AEI.
Outline of the talk

Why study the merger of binary neutron stars

Earlier (AEI) results for equal-mass NS binaries

role of the mass
role of the EOS

Preliminary results for unequal-mass NS binaries
(current activity on Mare Nostrum)
Double neutron star binaries exist in Nature

<table>
<thead>
<tr>
<th>Name</th>
<th>M₁/Mₗₚsun</th>
<th>M₂/Mₗₚsun</th>
<th>q=M₂/M₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1534+12</td>
<td>1.33</td>
<td>1.34</td>
<td>0.99</td>
</tr>
<tr>
<td>B2127+11C</td>
<td>1.36</td>
<td>1.35</td>
<td>0.99</td>
</tr>
<tr>
<td>B1913+16</td>
<td>1.44</td>
<td>1.38</td>
<td>0.96</td>
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<td>J0737-3039</td>
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<td>1.25</td>
<td>0.94</td>
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<tr>
<td>J1906+0746</td>
<td>1.35</td>
<td>1.26</td>
<td>0.93</td>
</tr>
<tr>
<td>J1829+2456</td>
<td>1.14</td>
<td>1.36</td>
<td>0.84</td>
</tr>
<tr>
<td>J1756-2251</td>
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<td>1.18</td>
<td>0.84</td>
</tr>
<tr>
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<td>1.11</td>
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<tr>
<td>J1518+4904</td>
<td>1.56</td>
<td>1.05</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Stairs 2004
Why study binary neutron star mergers?

**Reason #1:** Because they are among the most powerful sources of **gravitational waves** and could be the Rosetta stone in high-density nuclear physics (critical key to decipher the NS physics).
Why study binary neutron star mergers?

Reason #2:
Because their inspiral and merger could be behind one of the most powerful phenomena in the universe: short Gamma Ray Bursts (GRBs)

HST images of July 9, 2005 GRB taken 5.6, 9.8, 18.6 & 34.7 days after the burst (Derek Fox, Penn State University)

short GRB, artist impression, NASA
Still very crude but it can be improved: microphysics for the EOS, magnetic fields, viscosity, radiation transport,

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu} \quad \text{(field eqs: 6 + 6 + 3 + 1)} \]

\[ \nabla_\mu T^{\mu\nu} = 0 , \quad \text{(cons. en./mom.: 3 + 1)} \]

\[ \nabla_\mu (\rho u^\mu) = 0 , \quad \text{(cons. of baryon no: 1)} \]

\[ p = p(\rho, \epsilon, \ldots) . \quad \text{(EoS: 1 + ...)} \]

This is not yet astrophysics but our approximation to “reality”.

\[ \nabla^* F^{\mu\nu} = 0 , \quad \text{(Maxwell eqs.: induction, zero div.)} \]

\[ T_{\mu\nu} = T^{\text{fluid}}_{\mu\nu} + T^{\text{em}}_{\mu\nu} + \ldots \]
Use a conformal and traceless “3+1” formulation of Einstein equations

Gauge conditions: “1+log” slicing for lapse; hyperbolic “Gamma-driver” for shift

Use consistent configurations of “irrotational” binary NSs in quasi-circular orbit

Use 4th-8th order finite-differencing

Wave-extraction with Weyl scalars and gauge-invariant perturbations

HD/MHD eqs (www.whiskycode.org)

HRSC methods with a variety of approx Riemann solvers (HLLE, Roe, Marquina, etc.) and reconstructions (PPM, minmod, TVD, etc.)

Method of lines for time integration

Use excision if needed

Use of suitable techniques for constraining the magnetic field to be divergence-free

AMR with moving grids (www.carpetcode.org)
Previous results: equal mass initial models

All the initial models are computed using the **Lorene code** for unmagnetized binary NSs (Bonazzola et al. 1999; [www.lorene.obspm.fr](http://www.lorene.obspm.fr)).

<table>
<thead>
<tr>
<th>Model</th>
<th>$M_1 = M_2 (M_\odot)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>low-mass</td>
<td>1.4</td>
</tr>
<tr>
<td>high-mass</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Technical data for the simulations:
- polytropic EOS, ideal-fluid EOS
- outer boundary: $\sim 86 M_{\odot}$ (total ADM mass) or $\sim 1.6$
- 8 refinement levels; res. of finest level: $\sim 0.008 M_{\odot}$
- PPM for the reconstruction
- Marquina flux formula
- Runge Kutta (3rd-order)
- Initial separation: 45 or 60 km

Baiotti, Giacomazzo, Rezzolla (2008)
A hot, low-density torus is produced orbiting around the BH. This is what is expected in short GRBs.
Matter dynamics

high-mass binary

soon after the merger the torus is formed and undergoes oscillations
Matter dynamics

high-mass binary

soon after the merger the torus is formed and undergoes oscillations
Gravitational waveforms: polytropic EOS

high-mass binary

first time the full signal from the formation to a bh has been computed
The behaviour:

“merger \rightarrow \text{HMNS} \rightarrow \text{BH + torus}”

is general but only qualitatively

Quantitative differences are produced by:

- differences in the mass for the same EOS:
  a binary with smaller mass will produce a HMNS which is further away from the stability threshold and will collapse at a later time

- differences in the EOS for the same mass:
  a binary with an EOS allowing for a larger thermal internal energy (i.e. hotter after merger) will have an increased pressure support and will collapse at a later time
The HMNS is far from the instability threshold and survives for a longer time while losing energy and angular momentum. After ~ 25 ms the HMNS has lost sufficient angular momentum and will collapse to a BH.
Matter dynamics comparison

**High-mass binary**

Soon after the merge, the torus is formed and undergoes oscillations.

**Low-mass binary**

Long after the merger, a BH is formed surrounded by a torus.

_barmode instability low mass_
First time the full signal from the formation to a BH has been computed. Development of a bar-deformed NS leads to a long GW signal.
The HMNS is not close to the instability threshold and survives for a much longer time.
After the merger a BH is produced over a timescale **comparable** with the dynamical one. After the merger a BH is produced over a timescale **larger** or much larger than the dynamical one.
After the merger a BH is produced over a timescale comparable with the dynamical one. After the merger a BH is produced over a timescale larger or much larger than the dynamical one.

Reasonable to expect that for any realistic EOS, the GWs will be between these two extreme cases. GWs will work as *Rosetta stone* to decipher the NS interior.
### Unequal-mass NS binaries run on Mare Nostrum

<table>
<thead>
<tr>
<th>Model</th>
<th>$M_{\text{total}}$ ($M_\odot$)</th>
<th>$q$</th>
<th>$J$ (g cm$^2$/s)</th>
<th>$\nu_{\text{orbit}}$ (Hz)</th>
<th>$\rho_{\text{max}}$ (g/cm$^3$)</th>
<th>$M_i$ ($M_\odot$)</th>
<th>$\bar{r}_i$ (km)</th>
<th>$\bar{A}_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M3.4q0.70</td>
<td>3.371</td>
<td>0.70</td>
<td>$7.98 \times 10^{49}$</td>
<td>298.47</td>
<td>$1.28 \times 10^{15}$</td>
<td>1.390</td>
<td>14.63</td>
<td>0.851</td>
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<tr>
<td>M3.4q0.80</td>
<td>3.375</td>
<td>0.80</td>
<td>$8.36 \times 10^{49}$</td>
<td>303.62</td>
<td>$9.21 \times 10^{14}$</td>
<td>1.498</td>
<td>13.81</td>
<td>0.904</td>
</tr>
<tr>
<td>M3.4q0.91</td>
<td>3.404</td>
<td>0.91</td>
<td>$8.33 \times 10^{49}$</td>
<td>299.06</td>
<td>$7.58 \times 10^{14}$</td>
<td>1.625</td>
<td>13.10</td>
<td>0.927</td>
</tr>
<tr>
<td>M3.5q0.75</td>
<td>3.464</td>
<td>0.75</td>
<td>$8.40 \times 10^{49}$</td>
<td>300.84</td>
<td>$1.27 \times 10^{15}$</td>
<td>1.485</td>
<td>13.96</td>
<td>0.886</td>
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<tr>
<td>M3.7q0.94</td>
<td>3.680</td>
<td>0.94</td>
<td>$9.37 \times 10^{49}$</td>
<td>306.56</td>
<td>$9.75 \times 10^{14}$</td>
<td>1.779</td>
<td>12.00</td>
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<td>M3.6q1.00</td>
<td>3.558</td>
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<td>$8.92 \times 10^{49}$</td>
<td>303.32</td>
<td>$7.58 \times 10^{14}$</td>
<td>1.779</td>
<td>12.04</td>
<td>0.950</td>
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<td>M3.8q1.00</td>
<td>3.802</td>
<td>1</td>
<td>$9.85 \times 10^{49}$</td>
<td>309.70</td>
<td>$9.74 \times 10^{14}$</td>
<td>1.901</td>
<td>10.96</td>
<td>0.963</td>
</tr>
</tbody>
</table>

David Link (Diplom Arbeit, AEI, 2009)

**High-resolution** version of these models currently running (Link) along with equal- and unequal-mass **magnetized** NS binaries (Giacomazzo).
Scaling and run details

**Walltime** (CPU time, communication & I/O)
Benchmark: load per core constant (36^3 grid points/core; 3 AMR levels)
Ideal scaling: constant horizontal line. **Good** (but not perfect) scaling up to 256 cores.

(According to MN support: *the scalability showed for the code is quite good if we compare with other similar codes executed at MareNostrum.*)

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**Grid Setup:**
6 refinement levels
Outer boundary: 240 km
Grid spacing from coarsest to finest (km): 6.0, 3.0, 1.5, 0.75, 0.375, 0.1875
Size individual moving grids (coarsest-to-finest; km): 180, 120, 60, 30, 15.

**Grid points (finest level):**
\[(2 \times 15/0.1875)^3 = 4,096,000\]
**Grid points (coarsest level):**
\[(2 \times 240/6)^3 = 512,000\]

**Memory requirements:** ~170 GB of total memory usage (140 cores)

**Duration:** 140 cores. Average runtime ~260 hours ~36,400 CPU hours/run. (high-res runs ~10^5 CPU hours/run)
Animation of Model M3.4q0.70 (xy plane)

**Dynamics:**

Asymmetry of binary system apparent at t=0

Heavier star more compact.

Tidal disruption (tail) and angular momentum transport.

Massive accretion torus (10% more massive than equal-mass case).

Recoil velocity.
Dynamics:

Asymmetry of binary system apparent at $t=0$

Heavier star more compact.

Tidal disruption (tail) and angular momentum transport.

Massive accretion torus (10% more massive than equal-mass case).

Recoil velocity.

Recoil of the torus-black hole system.
Animation of Model M3.4q0.70 (xz plane)

**Dynamics:**

Tidal disruption (tail) and angular momentum transport.

Massive accretion torus (10% more massive than equal-mass case).
Compilation of main results

<table>
<thead>
<tr>
<th>Model</th>
<th>M_T/M_{\text{sun}}</th>
<th>M_T/M_{\text{total}}</th>
<th>r_e (km)</th>
<th>r_p (km)</th>
<th>Error (M_T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M3.6q1.00</td>
<td>0.001</td>
<td>0.1%</td>
<td>26</td>
<td>2.5</td>
<td>-</td>
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<tr>
<td>M3.7q0.94</td>
<td>0.006</td>
<td>0.4%</td>
<td>31</td>
<td>5.5</td>
<td>26.7%</td>
</tr>
<tr>
<td>M3.4q0.91</td>
<td>0.079</td>
<td>4.6%</td>
<td>51</td>
<td>15</td>
<td>6.3%</td>
</tr>
<tr>
<td>M3.4q0.80</td>
<td>0.120</td>
<td>7.1%</td>
<td>58</td>
<td>15</td>
<td>2.5%</td>
</tr>
<tr>
<td>M3.5q0.75</td>
<td>0.098</td>
<td>5.6%</td>
<td>66</td>
<td>15</td>
<td>1.3%</td>
</tr>
<tr>
<td>M3.4q0.70</td>
<td>0.132</td>
<td>7.9%</td>
<td>75</td>
<td>16</td>
<td>14.7%</td>
</tr>
</tbody>
</table>

- Resulting tori may be stable configurations.
- All considered models satisfy $u_t > -1$, and hence large fraction of torus material is bound.
- The more extreme $q$ the larger the equatorial and polar dimensions of the torus, $r_e$ and $r_p$, and the larger the final torus mass $M_T$, and the larger the recoil velocity of the system.
- Rough empirical relation shows that $M_T$ of up to $0.4M_{\text{sun}}$ might be feasible.
Simulations with idealised EOS have reached possibly the most complete description of BNSs from the inspiral, merger, collapse to BH.

GWs from BNSs are much complex/richer than from BBHs: can be the Rosetta stone to decipher the NS interior.

Unequal-mass BNS simulations show the formation of long-term stable tori with masses up to 0.4$M_{\text{sun}}$.

**Much remains to be done** to model realistically BNSs, both from a microphysical point of view (EOS, neutrino emission, etc) and from a macrophysical one (instabilities, etc.) **This poses not only a physical challenge but also a computational one.**